

DIGITAL FILTERING  
AND ACCELERATION PULSE INTERPRETATION

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## TOPICS

This presentation will address digital filtering of the CID data and a technique to analyze the acceleration data (fig. 1). Analog filtering was performed by electronic hardware (low pass analog filters) on board the aircraft to remove undesired high frequencies and to prevent errors due to sampling. Digital filtering is a computerized post-processing filtering technique that can be applied to the digitized time history data. Low pass, high pass, and notch filters can be mathematically simulated with this technique.

In order to validate acceleration traces, the integrated crash pulse velocity change was computed. Traces with velocity change outside the expected envelope were either rejected or set aside for additional study. Average accelerations were computed by dividing the primary impact velocity change by the pulse duration. An equivalent triangular pulse acceleration was computed by doubling the average acceleration and this value was compared to peak acceleration values. Selected traces were chosen that illustrate the effects of digital filtering for structure and dummy occupants. Other traces were chosen to illustrate the method used to obtain the average pulse acceleration and to triangularize the acceleration pulse.

- Digital filtering
  - Digital versus analog filtering
- Data interpretation
  - Peak and average accelerations
  - Velocity
  - Triangularizing the acceleration pulse
- Discussion of selected data traces

Figure 1

## SAE CLASS 60 FILTER

A low pass filter with frequency response that lies within the inner and outer limits shown by the solid straight lines is defined by SAE J211a (ref. 1) to be a class 60 filter. The digital filter chosen to smooth CID accelerometer data is shown on figure 2 by a dashed line. The digital filter is flat with no attenuation (0 decibels) to the cutoff frequency  $f_c$  and then rolls off until it attenuates all frequencies above the terminal frequency  $f_t$ . This digital filter with  $f_c = 10$  Hz and  $f_t = 188$  Hz will be designated henceforth as the CID 100-Hz digital filter.

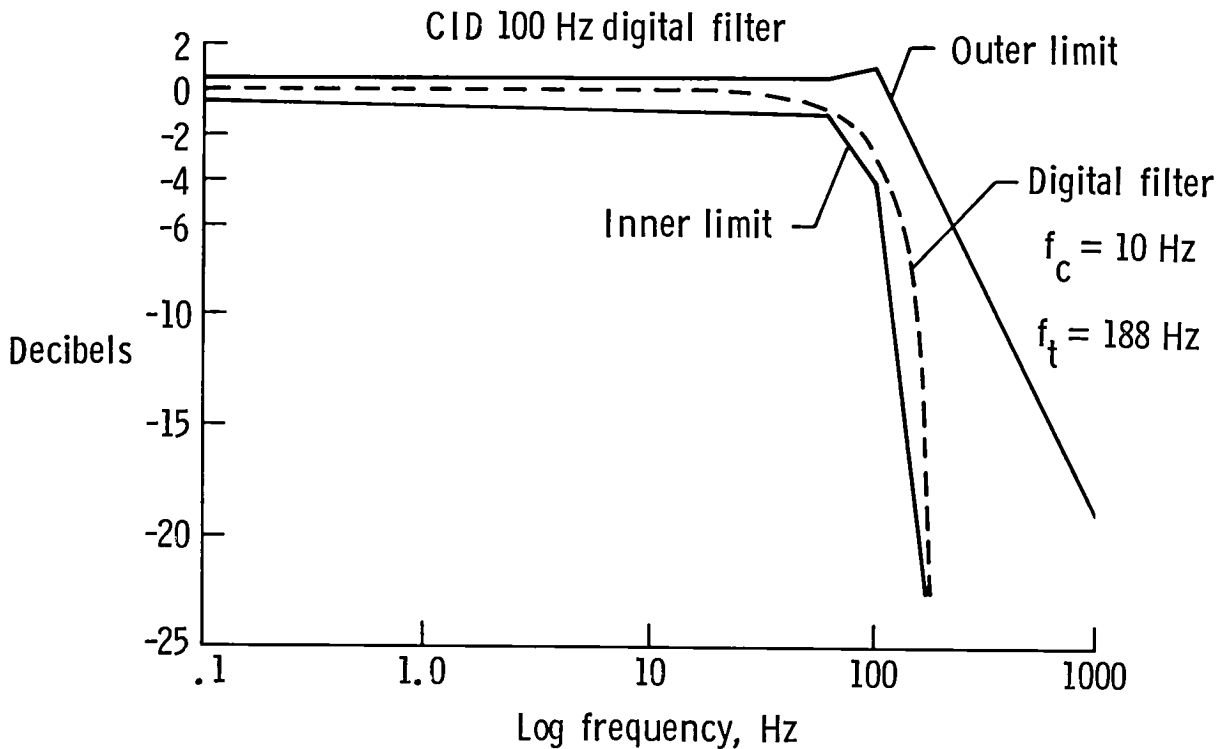


Figure 2

# DIGITAL FILTERING EXAMPLE

The upper trace in figure 3 illustrates the "raw" acceleration time history for channel 276 without any digital filtering. At the bottom left corner of the lower plot, the number 33731.000 is the starting time of the plot in seconds and corresponds to 9 hours 22 minutes and 11 seconds. The lower plot is a magnification of the 0.02 second period from 33731.60 to 33731.62 seconds. The circles are the original raw data points that were sampled at every 1000th of a second, and these raw data points are connected by straight line segments. (In the top plot the circles are left out and only the line segments are present.) The data points represented by circles were input into the 100-Hz digital filter program. For each circle or raw data point, a corresponding square digitally filtered point is shown in the bottom plot. In actual practice, the filtering program uses  $N$  raw data points before time  $t_n$  and  $N$  data points after time  $t_n$  to calculate one filtered value at  $t_n$ .

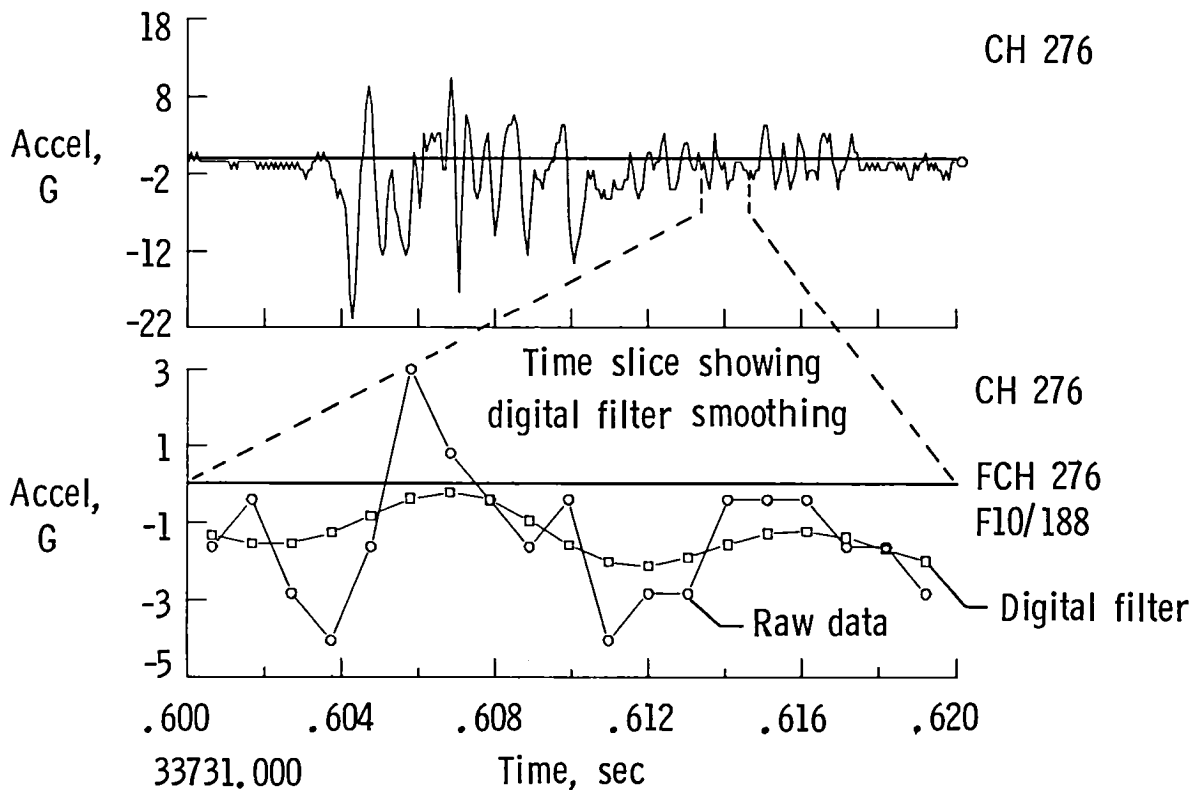


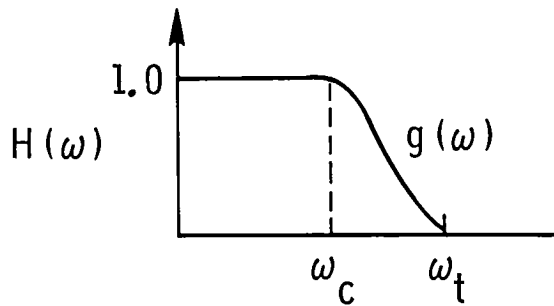
Figure 3

## DIGITAL FILTERING BACKGROUND MATHEMATICS

Figure 4 illustrates the low pass filter gain ratio  $H(\omega)$  in the frequency domain. The gain is one (no attenuation) for frequencies from 0 to the cutoff frequency  $\omega_c$  and then starts rolling off as defined by an arbitrary decreasing function  $g(\omega)$  with boundary conditions  $g(\omega_c) = 1$  and  $g(\omega_t) = 0$ . To transform the gain function to the time domain, the inverse Fourier transform  $h(t)$  must be evaluated.

### Low pass filter characteristics

- Frequency domain



$g(\omega) =$  arbitrary decreasing function  
with

Boundary conditions  $\begin{cases} g(\omega_c) = 1 \\ g(\omega_t) = 0 \end{cases}$

where  $\omega = 2 \pi f$

- Time domain

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} H(\omega) d\omega$$

Figure 4

## DIGITAL FILTERING BACKGROUND MATHEMATICS

For simplicity, let us assume the function  $g(\omega)$ , which defines the rolloff of the filter in the frequency domain, is a cosine function (ref. 2). Then the inverse Fourier transform  $h(t)$  can be evaluated in closed form as given in figure 5. Next assume the data points to be filtered are equally spaced with interval  $\Delta t$ . For this fixed  $\Delta t$ , let us evaluate an arbitrary  $2N + 1$  values of  $h(n \Delta t)$ . The  $n$ th normalized value of  $h(n \Delta t)$  is denoted by  $\bar{h}_n$  bar.

Evaluate Inverse Fourier Transform

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} H(\omega) d\omega \quad \text{with rolloff } g(\omega) \text{ a cosine function}$$

$$\text{then } h(t) = \frac{\pi}{2t} \left[ \frac{\sin \omega_t t + \sin \omega_c t}{\pi^2 - (\omega_t - \omega_c)^2 t^2} \right]$$

assume  $m$  equally spaced data points  $(d_1, t_1), (d_2, t_2) \dots (d_m, t_m)$

where  $t_n = n\Delta t$

Evaluate  $h(t)$  to obtain  $2N + 1$  coefficients for a given fixed  $\Delta t$

Denote  $\bar{h}_n$  the  $n^{\text{th}}$  normalized coefficient

$$\bar{h}_n = \frac{h_n}{\sum_{-N}^N h_\ell}$$

Figure 5

## DIGITAL FILTERING BACKGROUND MATHEMATICS

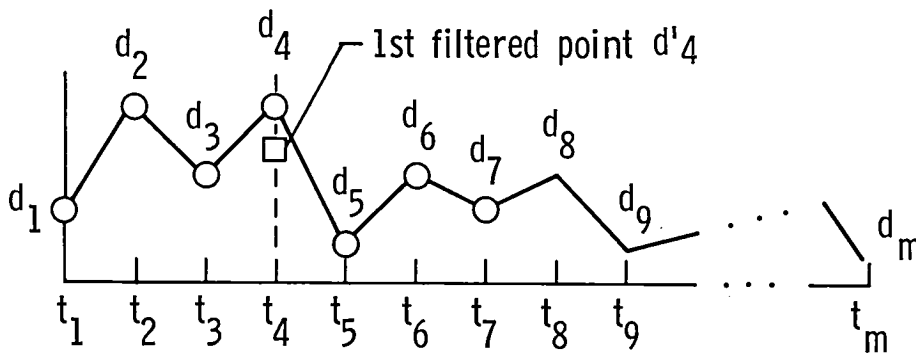
The  $N+1$  filtered data point is denoted in figure 6 as  $d'_{N+1}$  and is given by a summation which is a Fourier convolution. Note from the formula (figure 6) that to calculate the first filtered data point  $N$  data points are used before the first filtered value and  $N$  data points are used after the first filtered value. In the simple example where  $N = 3$ , there are seven ( $2N+1$ ) terms in the formula to calculate each filtered data value. The first filtered value  $d'_4$  is generated by using points  $d_1$  to  $d_7$  in the formula. To calculate the second filtered value  $d'_5$  the same process is used by taking the seven points  $d_2$  through  $d_8$  each multiplied by the proper  $\bar{h}$ .

The integer  $N$  needed for 1 percent accuracy can be computed from the last formula where  $f_t - f_c$  is the rolloff frequency region.

$$d'_{N+1} = \sum_{-N}^N \bar{h}_n d_{N+n+1} = \bar{h}_{-N} d_1 + \dots + \bar{h}_0 d_{N+1} \dots + \bar{h}_N d_{2N+1};$$

$$\text{where } \bar{h}_{-i} = \bar{h}_i$$

$$\text{for example if } N = 3 \quad d'_4 = \bar{h}_{-3} d_1 + \dots + \bar{h}_0 d_4 + \dots + \bar{h}_3 d_7$$



$$\text{For 1\% accuracy we require } N\Delta t (f_t - f_c) \geq 2$$

Figure 6

## ERROR ANALYSIS OF CID 100-Hz DIGITAL FILTER

To evaluate the accuracy of the  $2N+1$  terms used to represent the gain function, one can expand  $H(f)$  as a Fourier series using the formula given in figure 7. For the CID digital filter,  $F_1$ , the cutoff frequency, was chosen to be 10 Hz,  $F_2$ , the termination frequency, was chosen to be 188 Hz, and  $N$  was arbitrarily chosen to be 150. The gain function  $H(f)$  plotted on the left can be seen to accurately represent the intended filter. Using the accuracy check formula at the bottom of figure 7,  $N(\Delta t) \cdot \text{rolloff}$  is equal to 53.7 for a sample rate of 500. Since this value is greater than two, the error of the filter is less than one percent. For a sample rate of 1000 a digital filter of 100-Hz can also be generated with error less than one percent. Since CID data were sampled at both 500 and 1000 samples per second, two 100-Hz digital filters had to be programmed, one for each sample rate.

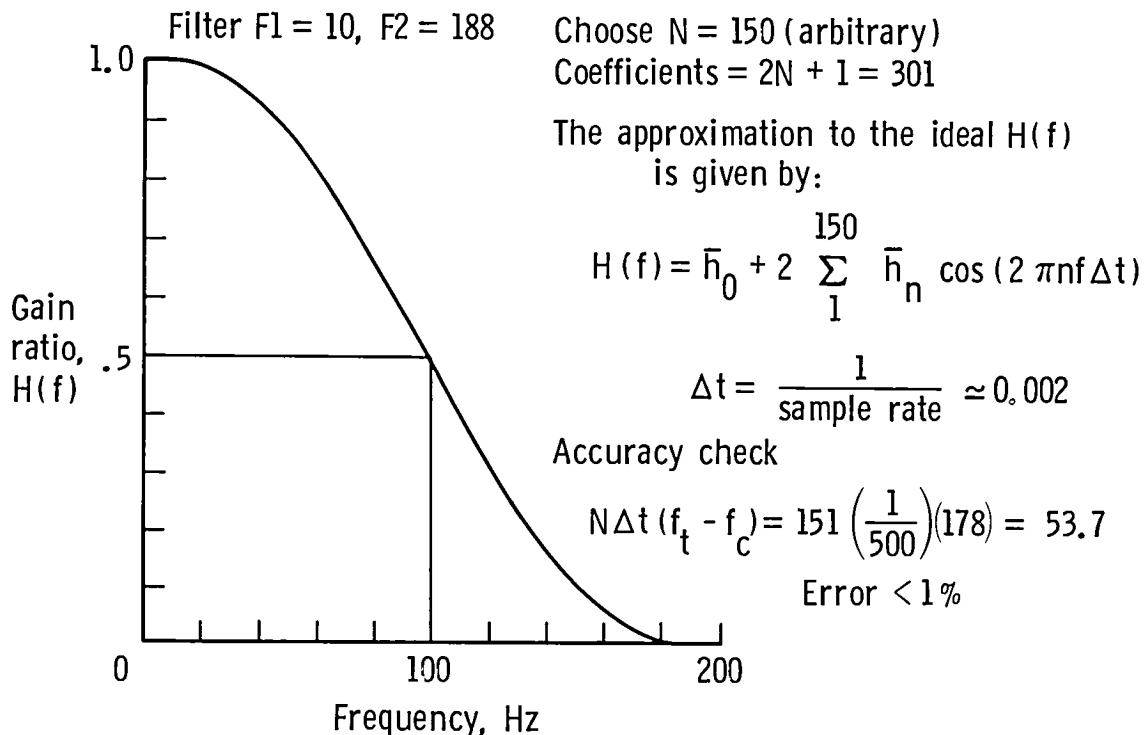


Figure 7



### EXAMPLE OF DIGITAL FILTER WITH ERROR EXCEEDING 1 PERCENT

Figure 8 illustrates an example of a digital filter with  $N = 150$  that demonstrates undesirable overshoot and undershoot from the ideal gain function. In this example if one calculates  $N(\Delta t)(\text{rolloff})$  using the accuracy check formula, a value of only 0.375 is obtained. Since this value is less than 2, the error would exceed 1 percent. To obtain the desired accuracy, either  $N$  or the rolloff region would need to be increased.

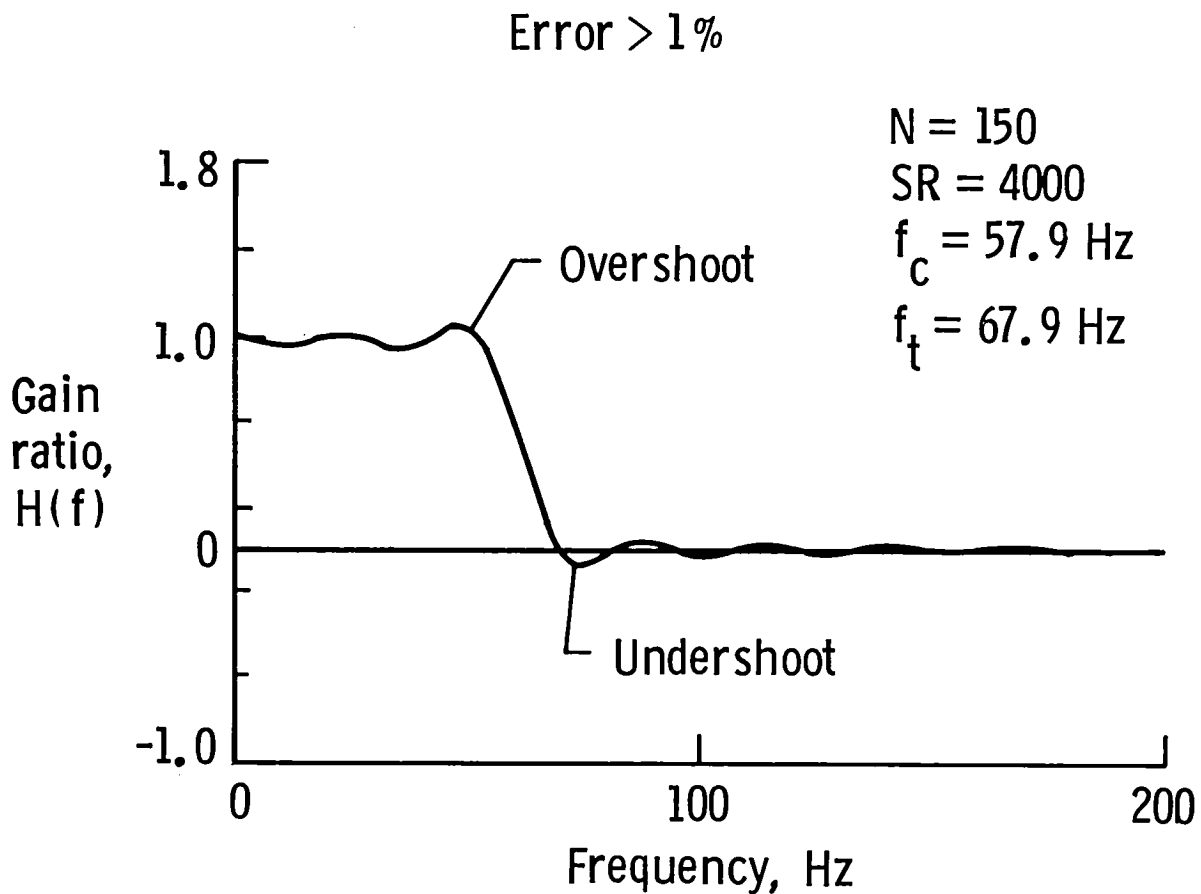


Figure 8

# PILOT FLOOR ACCELERATION NORMAL DIRECTION

The pilot floor acceleration data (normal to the floor) were sampled at 500 samples per second and were taken from an accelerometer attached to the floor at body station 228. The two plots shown in figure 9 are from channel 1 for the first second after engine number one on the left wing contacted the ground. The top plot is the raw data as recorded with no post-crash digital filtering. The second plot shows the data after passing through the 100-Hz digital filter discussed previously. The filter attenuates the high frequencies and smoothes the digital data.

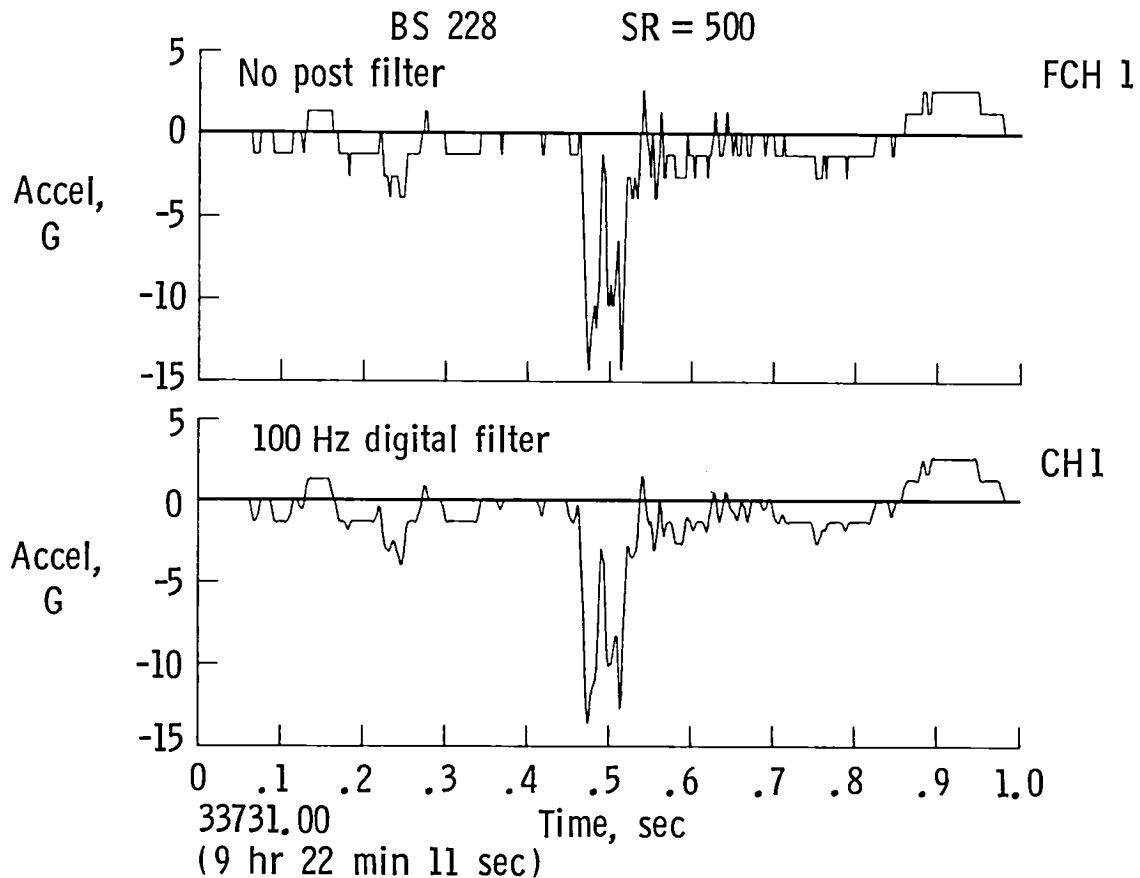


Figure 9

## FLOOR TRACK ACCELERATION NORMAL DIRECTION

The two plots in figure 10 show the floor track acceleration for the first instrumented frame at body station 400. For this example, the sample rate is 1000. Again the top plot is the data with no post filter and the bottom plot is for the data passed post test through the 100-Hz digital filter with  $F1 = 10$  Hz and  $F2 = 188$  Hz. These data were taken from CID channel 276.

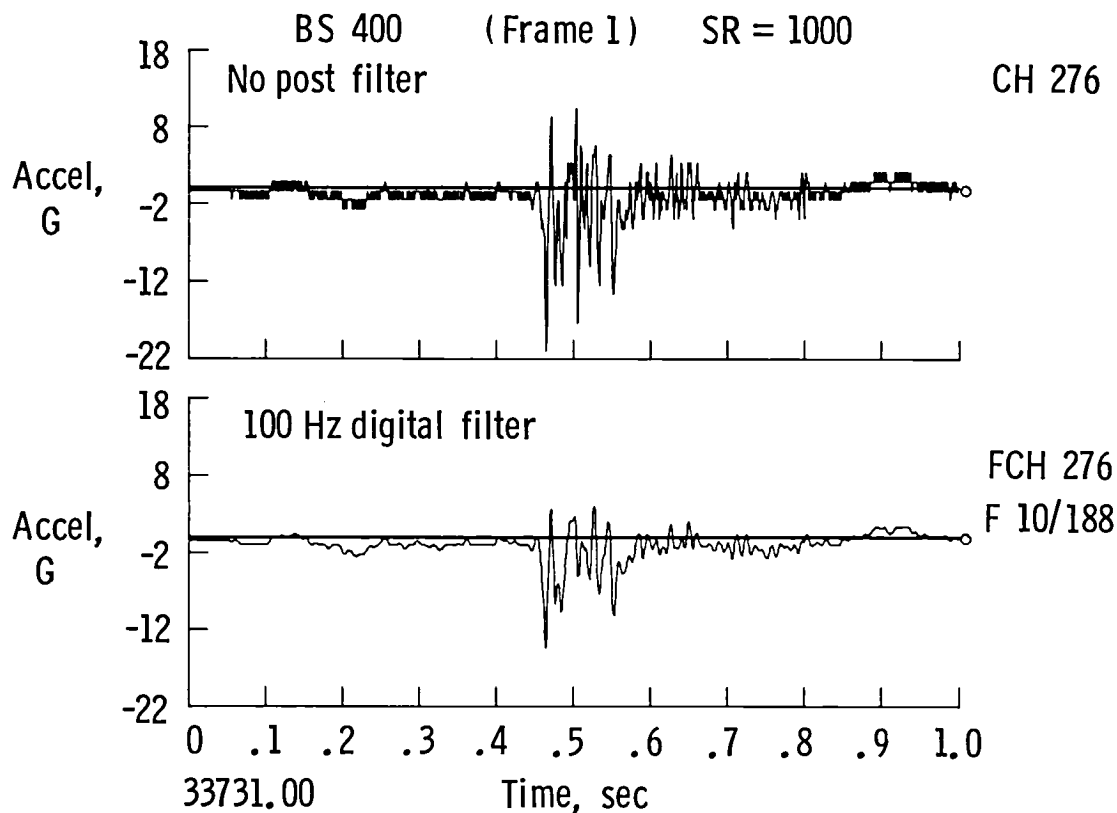


Figure 10

## FLOOR ACCELERATION NORMAL FOR FRAME 2

These two acceleration traces (CID channel 120) were taken at body station 540. As we move back in the airplane, the acceleration levels become lower and lower and the digital nature of the data is more pronounced. In figure 11 the smoothing effect of the 100-Hz digital filter can clearly be seen. In the top plot, the acceleration data are quite "stairstep looking" due to the resolution of the 8-bit digital data over the  $\pm 150$  G range required for the normal (vertical) direction. The lower plot, which is the same data filtered with the 100-Hz digital filter, has all the low-frequency characteristics of the raw data without the digital "stairsteps."

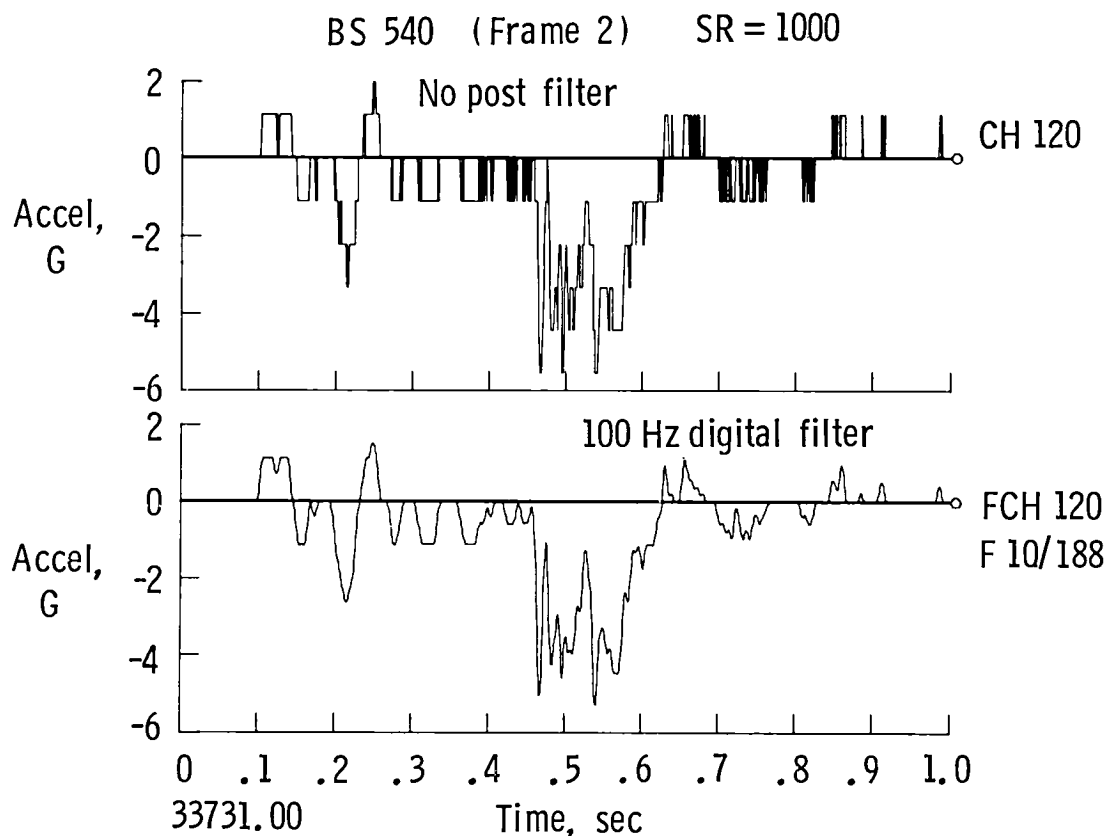


Figure 11

# DUMMY 14B PELVIS ACCELERATION NORMAL

The pelvis acceleration along the spineward direction for the center dummy in the left seat in row 14 is shown in the two traces in figure 12. The sample rate is 1000 per second. Again the 100-Hz filter removes the high-frequency noise and cleans up the trace quite well.

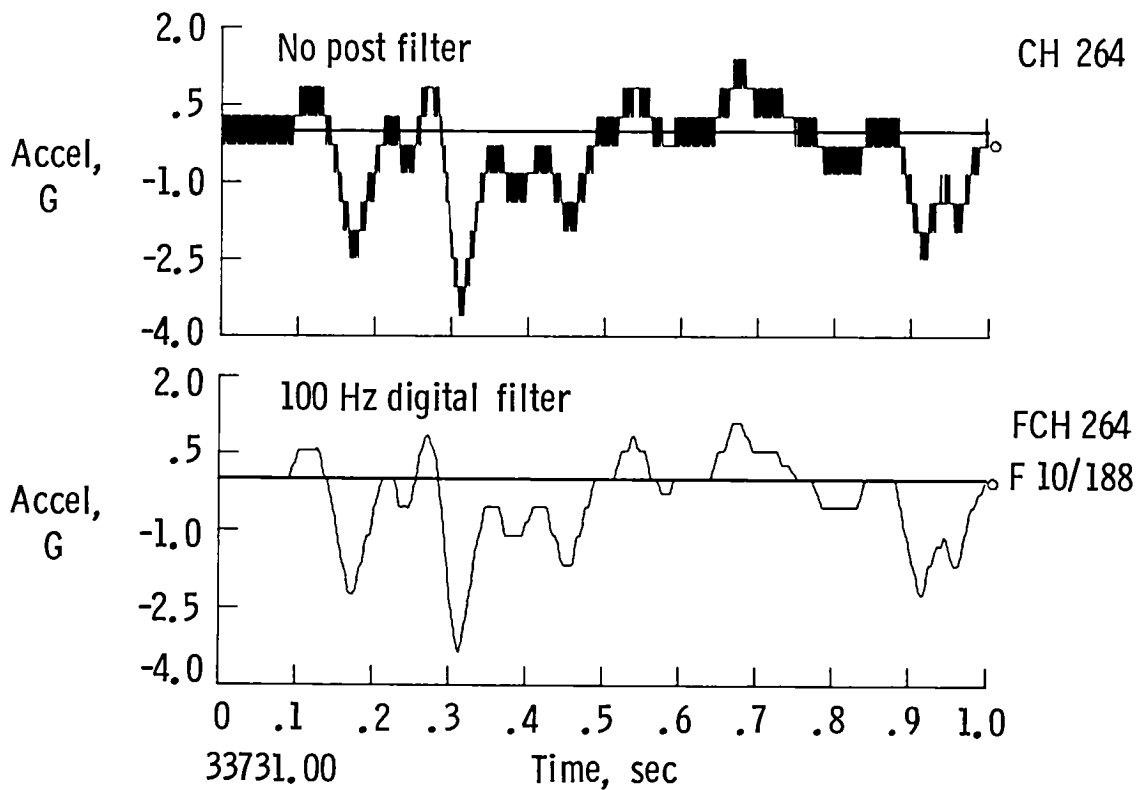


Figure 12

### DUMMY 3E PELVIS ACCELERATION LONGITUDINAL DIRECTION

All longitudinal accelerations measured on the aircraft and dummies were relatively low. Since the resolution of the 8-bit system was approximately 1 G per count for this particular channel (an 8-bit system can have from 0 to 255 counts), the actual acceleration trace is only bounded by the data. The digital filter does a reasonable job; however, it cannot make up for the loss of resolution in this case (fig. 13).

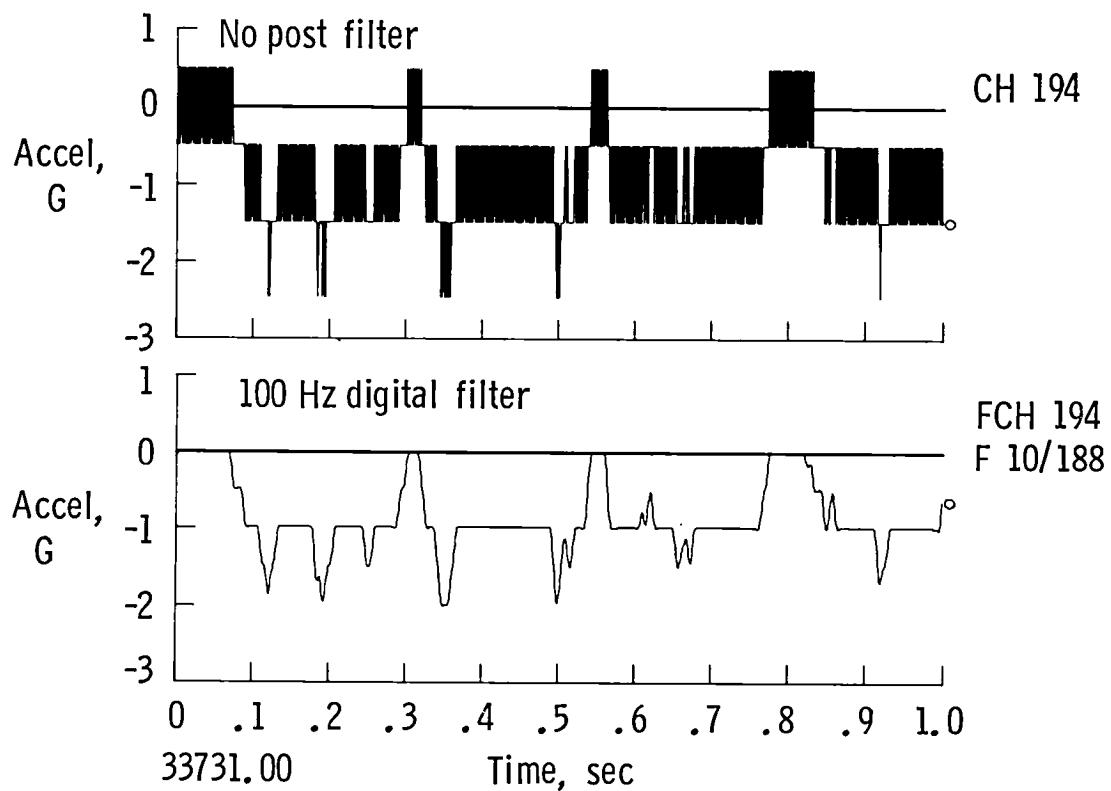


Figure 13

# VERTICAL FLOOR ACCELERATION AND INTEGRATED VELOCITY

The top plot in figure 14 is of the normal acceleration of the floor at body station 540. The bottom plot shows the velocity curve obtained by integrating the top acceleration plot. The dominant acceleration pulse from 0.48 to 0.62 seconds in time occurs when the fuselage impacts the ground. The acceleration from 0 to 0.48 seconds is from the left wing impact. Notice that the total vertical velocity change is approximately 18 ft/s, but that the velocity change is composed of almost 4 ft/s taken out by the wing and about 14 ft/s taken out by the fuselage. The average acceleration for the fuselage impact can be computed from delta V of 14 ft/s divided by delta T of 0.14 seconds which when expressed in G-units is 3.1 G's. If the acceleration from 0.48 to 0.62 seconds were a triangular pulse, then the peak would be twice the average or 6.2 G's. For this example, twice the average acceleration and the peak acceleration directly from the plot are nearly the same. The average acceleration and twice the average acceleration are useful indicators in data interpretation of crash impact severity.

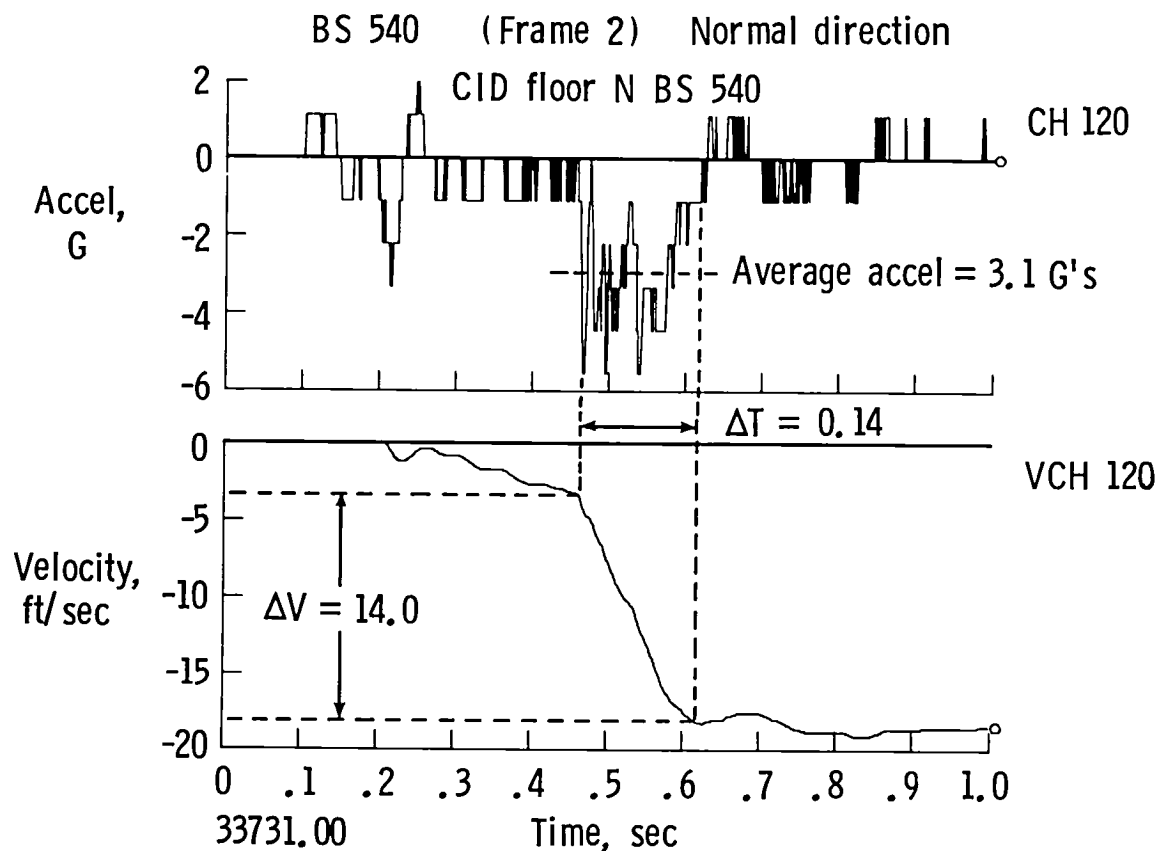


Figure 14

## PILOT FLOOR ACCELERATION AND INTEGRATED VELOCITY (LONGITUDINAL)

Using the same technique as in figure 14, the longitudinal acceleration of the pilot floor is analyzed for the 0.13 second pulse shown in figure 15. Note that the average acceleration is only 0.84 G in this case. Twice the average acceleration, which would correspond to a 0.13 second triangular pulse, would only have a peak of 1.68 G's. The peak G taken from the top trace is about 6 G's, but it is of very short duration. The average acceleration or peak triangularized acceleration has more meaning in this case and can be compared with other locations in the aircraft with better results.

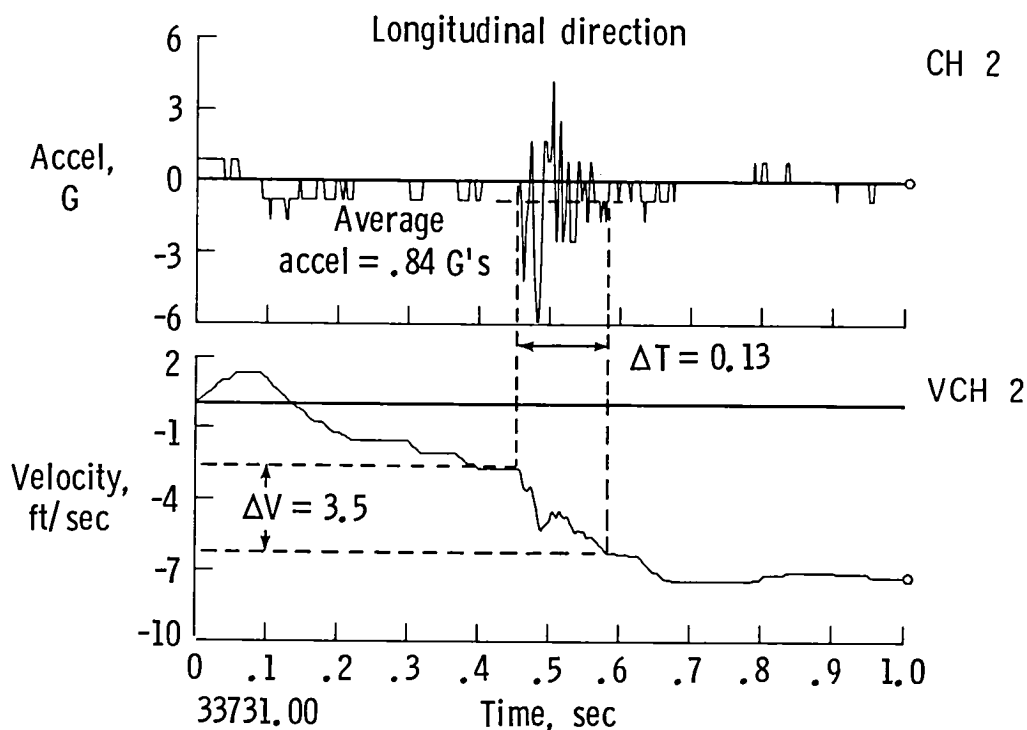


Figure 15



# SELECTED CID CHANNELS - PRIMARY IMPACT

The primary impact is defined to be the impact from the time the left wing contacted the ground (9 hrs 22 min 11 sec or 33731.0 sec) until the rear of the fuselage made contact (about one second). The wing cutter impact which occurred later will not be analyzed in this discussion. A cross section of data from 51 representative CID channels was analyzed and distributed at this conference. Figure 16 shows the format of the 34 fuselage accelerometer channels that were analyzed. For each channel the location of the accelerometer, the direction, the body station, the peak acceleration value, time the peak occurred, the velocity change of the main pulse, the average acceleration of the main pulse, twice the average acceleration, the sample rate, and the analog filter on board the aircraft are all given. In addition, one-second traces from the time of left wing contact are shown.

## Fuselage accelerometer data:

Cha	Location	DIR	BSTA	Peak val G	Peak time S	$\Delta V$ ft/s	$\Delta T$ S
2	Pilot floor	L	228	6.10	0.48	3.51	0.13
1	Pilot floor	N	228	14.30	0.48	17.60	0.08
3	Pilot floor	T	228	4.00	0.14	10.00	0.12
276	FR #1 floor rt track	N	400	14.50	0.47	14.00	0.12
286	Frame 1 top	N	400	16.20	0.48	15.20	0.12

Cha	Location	AVEACC G	2 x AVE G	Sample rate Hz	Analog filter Hz
2	Pilot floor	0.84	1.68	500	100
1	Pilot floor	7.29	14.58	500	100
3	Pilot floor	2.59	5.18	1000	100
276	Fr # 1 floor ft track	3.62	7.25	1000	100
286	Frame 1 top	3.93	7.87	1000	180

Figure 16

# SELECTED CID CHANNELS - PRIMARY IMPACT

Figure 17 illustrates the format of the 14 selected channels of wing and pylon accelerometer data. Some of the velocity changes and other analysis are not given because of accelerometer overranging that occurred for some channels. All of the 14 selected channels were sampled at 500 samples per second and were filtered on board with 100-Hz analog filters.

Wing and pylon accelerometer data:

Sample rate: 500 per second

Analog filter: 100 Hz

Cha	Location	DIR	BSTA	Peak val G	Peak time S	$\Delta V$ ft/sec	$\Delta T$ S	AVEACC G	$2 \times AVE$ G
332	L Engine front pylon	N	XXXX	80.00	0.15	---	---	---	---
339	R Engine front pylon	N	XXXX	5.10	0.62	14.40	0.19	2.35	4.71
336	L Engine rear pylon	N	XXXX	162.70	0.12	---	---	---	---
342	R Engine rear pylon	N	XXXX	28.70	0.62	---	---	---	---
184	L Wing F spar in	L	806	19.10	0.11	---	---	---	---

Figure 17

# LAP BELT LOAD CELL DATA

Figure 18 presents the lap belt load cell data for three selected channels. Since load cells have lower frequency response than accelerometers, a 50-Hz digital filter was chosen to filter the lap belt load cell data. The digital filter for load cell data attenuates more of the high frequencies with F1 = 6 Hz, F2 = 94 Hz, and - 3 dB attenuation for 50 Hz. The loads measured in the lap belts were very low because of the low longitudinal accelerations in the crash.

Load cells digitally filtered post crash (f1 = 6 Hz, f2 = 94 Hz, -3db at 50 Hz)

Cha	Location	BSTA	Peak val LB	Peak time S	Sample analog Rate 1/S	Filter Hz
267	Dummy 14B left belt	1220	80.00	0.98	500	60
103	Dummy 14E left belt	1220	43.00	0.54	500	100
104	Dummy 14E rt belt	1220	60.00	0.54	500	100

Figure 18

## SUMMARY

In summary, the post processing digital filter is quite effective for removing unwanted high frequency and noise without distorting the signal. In addition, digital filtering is effective for smoothing low-level low resolution digital data where the digital "stairstep" phenomena are pronounced.

Integrating the acceleration data to obtain velocity traces is quite useful. The velocity change provides a check on the validity of the acceleration trace. (Zero offsets in acceleration must be removed before integrating.) Average accelerations can be obtained from the velocity trace by dividing the change in velocity for an acceleration pulse by the pulse duration. One can obtain the peak of an equivalent triangular pulse by doubling the average acceleration (fig. 19).

- Digital filter effective for
  1. Removing unwanted high-frequency vibrations and noise
  2. Smoothing digital data
- Integrated acceleration (velocity) useful for
  1. Obtaining velocity changes
  2. Establishing validity of acceleration traces
  3. Obtaining average accelerations
  4. Triangularizing the acceleration pulse

Figure 19

#### REFERENCES

1. Society of Automotive Engineers: Instrumentation for Impact Tests. SAE J211a, revised 1971.
2. Graham, Ronald J.: Determination and Analysis of Numerical Smoothing Weights. NASA TR R-179, 1963.